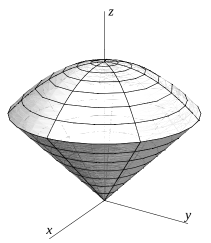
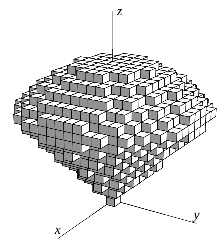
Triple Integrals

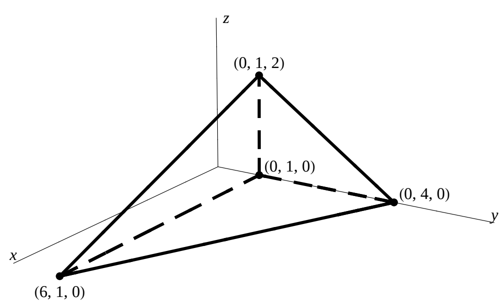




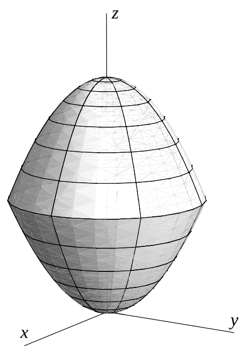
1. (a) If $\mathcal{U}$ is any solid (in space), what does the triple integral $\iiint_{\mathcal{U}} 1\ dV$ represent? Why?

(b) Suppose the shape of a solid object is described by the solid $\mathcal{U}$, and $f(x, y, z)$ gives the density of the object at the point (x, y, z) in kilograms per cubic meter. What does the triple integral $\iiint_{\mathcal{U}} f(x, y, z)\ dV$ represent? Why?

2. Let $\mathcal{U}$ be the solid tetrahedron bounded by the planes $x = 0$, $y = 1$, $z = 0$, and $x + 2y + 3z = 8$. (The vertices of this tetrahedron are $(0, 1, 0)$, $(0, 1, 2)$, $(6, 1, 0)$, and $(0, 4, 0)$). Write the triple integral $\iiint_{\mathcal{U}} f(x, y, z)\ dV$ as an iterated integral.



3. Let $\mathcal{U}$ be the solid enclosed by the paraboloids $z = x^2+y^2$ and $z = 8-(x^2+y^2)$. (Note: The paraboloids intersect where $z = 4$.) Write $\iiint_{\mathcal{U}} f(x,y,z)\, dV$ as an iterated integral in the order $dz\ dy\ dx$.



4. In this problem, we'll look at the iterated integral $\int_0^1 \int_0^z \int_{y^2}^1 f(x,y,z)\, dx\ dy\ dz$.

   (a) Rewrite the iterated integral in the order $dx\ dz\ dy$.

   (b) Rewrite the iterated integral in the order $dz\ dy\ dx$.

5. Let $\mathcal{U}$ be the solid contained in $x^2+y^2-z^2 = 16$ and lying between the planes $z = -3$ and $z = 3$. Sketch $\mathcal{U}$ and write an iterated integral which expresses its volume. In which orders of integration can you write just a single iterated integral (as opposed to a sum of iterated integrals)?